Predicting Thaw Degradation in Algid Climates along Highway Embankments using a Boundary Element Method

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1. INTRODUCTION

The Complex Variable Boundary Element Method or CVBEM is used to develop a computer model for estimating the location of the freezing front in soil-water phase change problems. The CVBEM based computer program is based on the following major assumptions:

(i) the problem is two-dimensional,

(ii) the entire soil system is homogeneous and isotropic,

(iii) the problem thermal boundary conditions are constant values of temperature (or stream function),

(iv) soil-water flow effects are neglected (the problem is strictly geothermal),

(v) all heat flow from the freezing front is within the control volume; there is no heat flux associated with the freezing front from exterior of the control volume,

(vi) the freezing front movement is sufficiently slow such that heat flux along the moving boundary can be determined by assuming steady state heat flow conditions.

The CVBEM is used to model the thermal regime of the soil system. The theory and development of the CVBEM is given in Hromadka (1984). Because the numerical technique is a boundary integral approach, the control volume thermal regime is a boundary integral approach, the control volume thermal regime is modeled with respect to the boundary values and, therefore, the computer program data entry requirements are significantly less than that usually required of domain methods such as finite-differences or finite-elements.

Soil-water phase change along the freezing front is modeled as a simple balance between computed heat flux and the evolution of soil-water volumetric latent heat of fusion. Because of the dominating effects of phase change of water in the heat balance equations, the governing heat transport equations are simplified to assuming steady state heat transport conditions above and below the freezing front. The CVBEM is applied independently in the thawed and in the frozen regions of the problem domain, with heat flux along the freezing front calculated by the CVBEM as the stream function that is conjugate to the potential function (temperature). Freezing and thawing is modeled as the net balance between the freezing and thawing heat flux over small modeling time steps. To model the displacement of the freezing front, the computer model provides two options:

(i) displace the freezing front coordinates with respect to changes in the y-coordinates only.
(ii) displace the freezing front coordinates with respect to a vector normal to the freezing front boundary.

OBJECTIVES OF DOCUMENTATION REPORT

The objectives of the Documentation report are:

(1) Provide background information regarding the CVBEM and the soil-water phase change model used in program CVBFR1.
(2) Provide documentation for the data entry sequence associated with program CVBFR1.
(3) FORTRAN computer code as an Appendix.

COMPUTATIONAL MATHEMATICAL MODEL DESCRIPTION

The problem domain under study is assumed to have a well-defined freezing front where soil water arriving at the freezing front location due to soil water saturated and unsaturated transport is subject to isothermal phase change. Further phase change effects beyond phase change of arriving soil water at the front, results in movement of the freezing front. Such displacement of the front redefines the problem heat transport domains of the thawed and frozen soil regions, respectively (Figure 1). Figure 2 depicts the relative vicinity of the frozen soil located above the freezing front and the thawed soils located below the front. In the model, an arbitrary temperature of 0.1 degrees centigrade is assumed applicable for the thawed oils that are subject to phase change due to periodic freezing and thawing cycles. The Complex Variable Boundary Element Method ("CVBEM") is the boundary element technique used to develop heat transport quantities along the freezing front, where phase change effects result in slow rates of displacement of the freezing front. Under such conditions, the Laplace equation is modeled in each of the thawed and frozen soil regions, respectively. Figure 3 shows a typical CVBEM modeling node layout. As soil water freezes or thaws, the location of the freezing front changes, with displacement occurring orthogonal to the freezing front contour (Figure 4). Firgures 5 and 6 shows the CVBEM model boundary value problem specification used for each of the thawed and frozen soil regions, respectively. The relationship between freezing front displacement and net heat flux at the front is depicted in Figure 7. Figure 8 shows details of the freezing front displacement geometric algorithm used. To validate the CVBEM model, comparison with a two-dimensional finite element method model of a roadway embankment in algid soils is made, with the FEM model discretized as shown in Figure 9. Figures 10 and 11 show the model comparison sections between the FEM and CVBEM models tested.

SUMMARY

Ongoing work on the described project is to be completed during the summer 2016 session. The funding for the project and coordination is through the AIAD program at West Point and is an element of the computational engineering mathematics theme at the Department of Mathematical-Sciences. This particular project topic involves aspects of both engineering-mathematics and computational geosciences.

REFERENCES

1. Johnson, N. Anthony, Carroll, M., Jones, L., Papas, N., Thomasy, C., Hromadka II, T.V., Horton, S., Whitley, R. and Johnson, M., 2013, A Computational Approach to Determining CVBEM Approximate Boundaries; Engineering Analysis with Boundary Elements, published online April 2014, Vol. 14, Pgs. 83-89.
2. Johnson, A. and Hromadka II, T.V., 2014, Modeling Mixed Boundary Conditions in a Hilbert Space with the Complex Variable Boundary Element Method (CVBEM), Methods X, ELSEVIER, published online August 2015, Volume 2, Pgs. 292-305.
3. Johnson, A.N., Hromadka II, T.V. and Horton, S.B., The Complex Variable Boundary Element Methods (CVBEM) for Mixed Boundaries, Presented at Joint Math Meetings 2015, San Antonio Texas, January 2015.

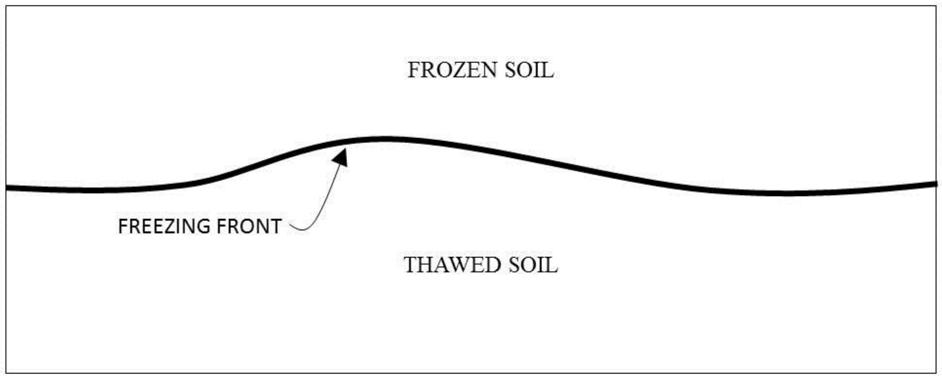

FIGURE 1. FREEZING FRONT LOCATION WITHIN MODEL DOMAIN.

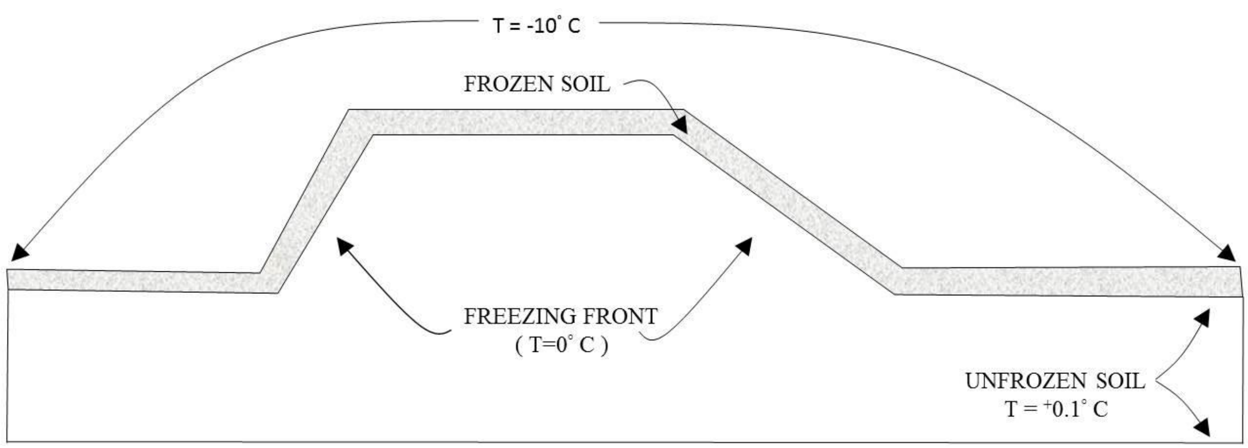

FIGURE 2. HEAT TRANSPORT ACCOUNTING ELEMENTS FOR PHASE CHANGE OF SOIL WATER.

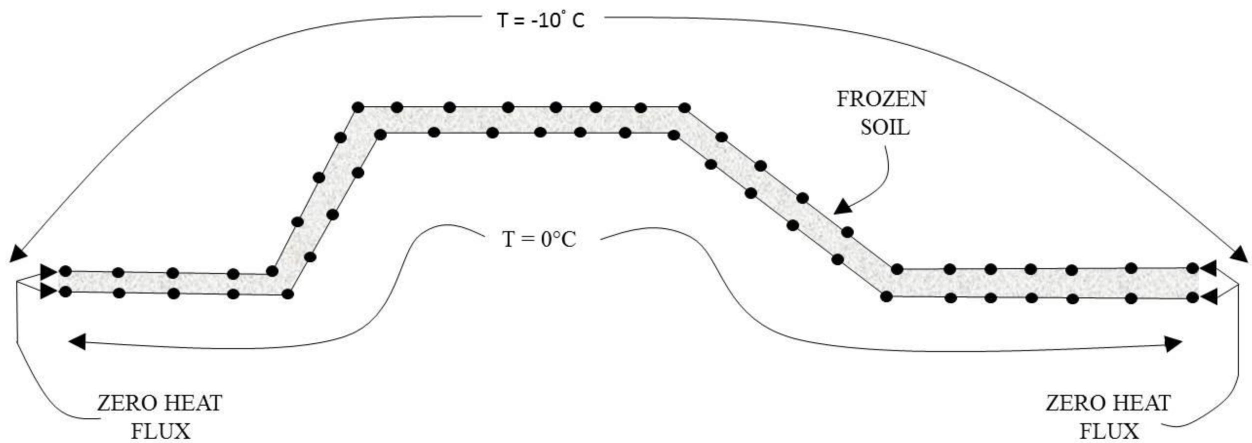

FIGURE 3. BOUNDARY ELEMENT METHOD NODAL POINT DEFINITION ON PHASE CHANGE REGION BOUNDARIES.

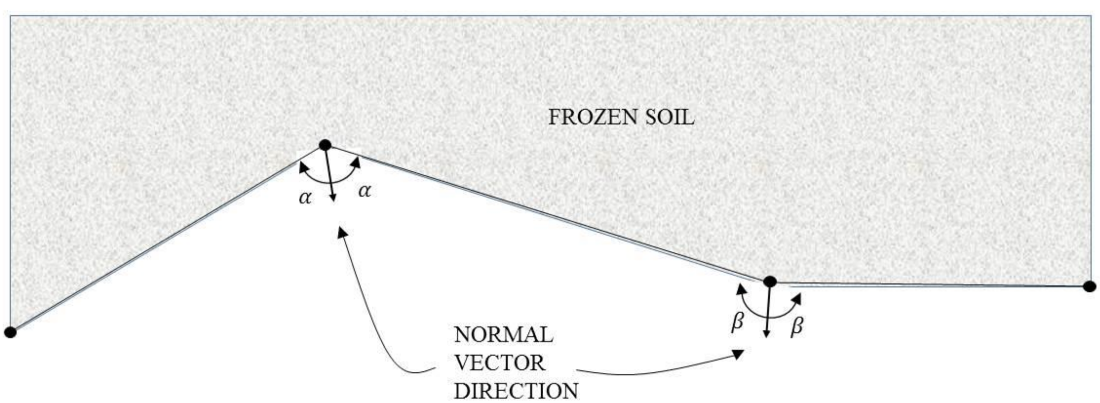

FIGURE 4. NORMAL VECTOR ORIENTATION WITH PHASE CHANGE SOIL REGION.

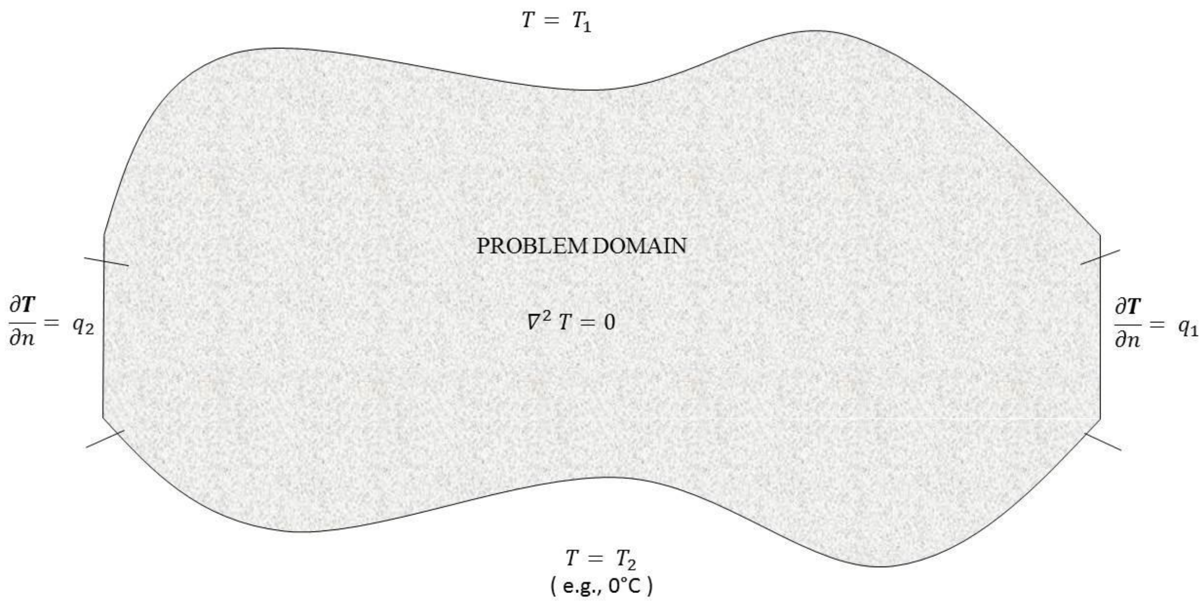

FIGURE 5. HEAT TRANSPORT BOUNDARY VALUE PROBLEM DEFINITION.

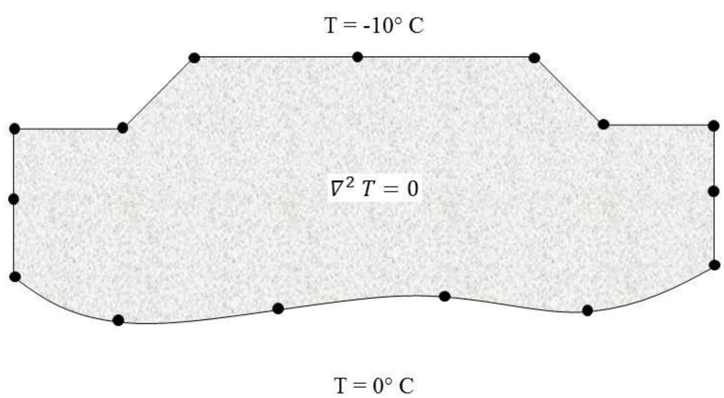

FIGURE 6. BOUNDARY ELEMENT NODE DEFINITION ON HEAT TRANSPORT MODEL DOMAIN BOUNDARIES.

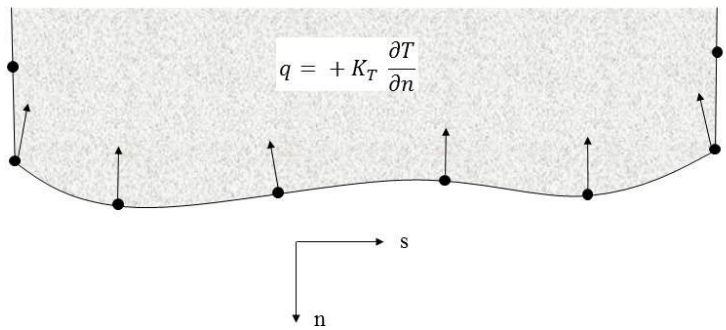

FIGURE 7. MOVING BOUNDARY ALGORITHM DEPICTION FOR PHASE CHANGE MODEL.

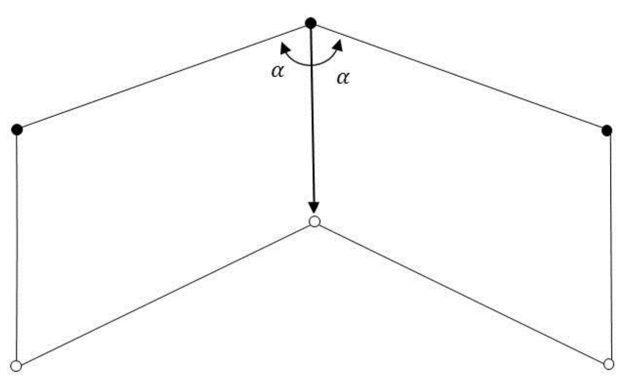

FIGURE 8. MOVING BOUNDARY DISPLACEMENT SCHEME DURING PHASE CHANGE.

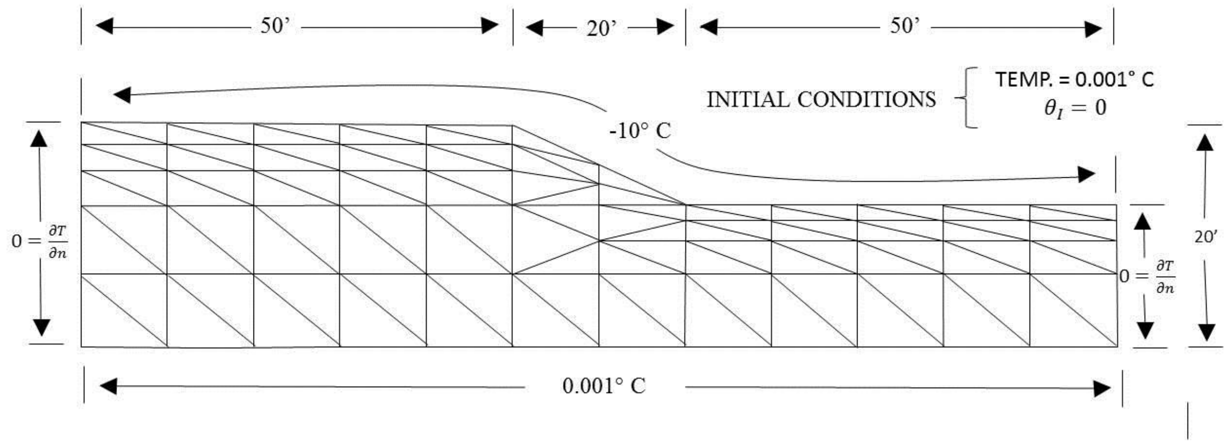

FIGURE 9. FINITE ELEMENT MODEL DISCRETIZATION FOR COMPARISON TO BOUNDARY ELEMENT MODEL.

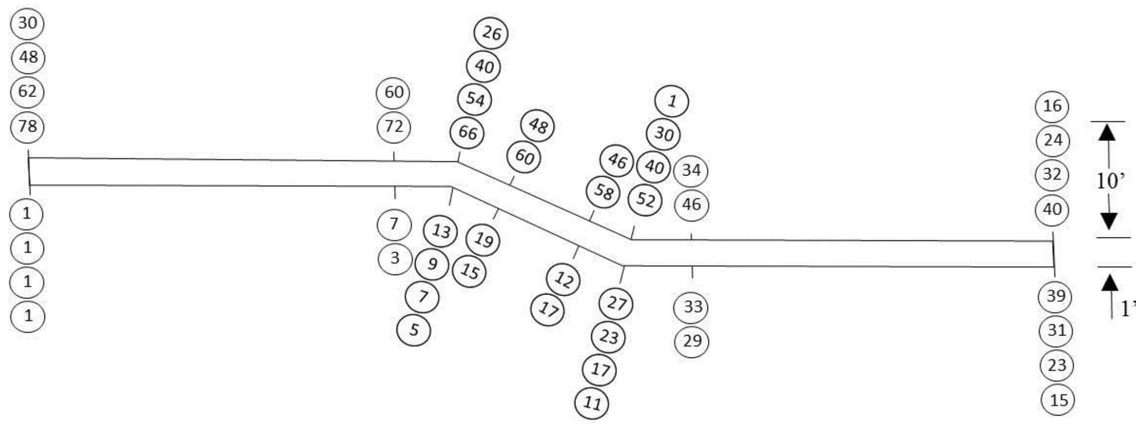

FIGURE 10. BOUNDARY ELEMENT NODE NUMBERING SCHEME.

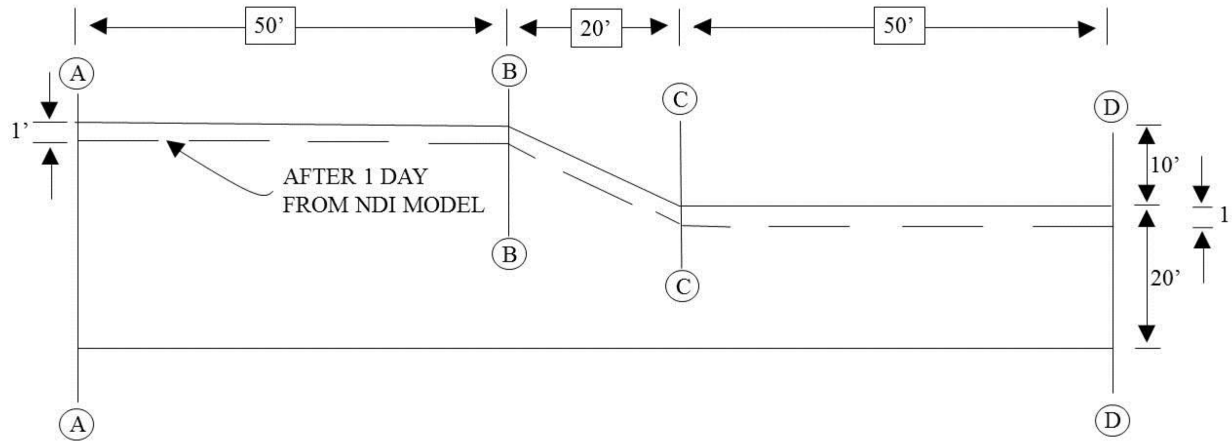

FIGURE 11. COMPARISON SECTIONS BETWEEN FEM MODEL AND BEM MODEL.