Cumulative Departure Model of the Cryosphere During the Pleistocene— an Application in Computational Engineering Mathematics

A TALK ON PAPERS, PROGRAMS AND PARTNERSHIPS


Technical Session #2
September, 2016

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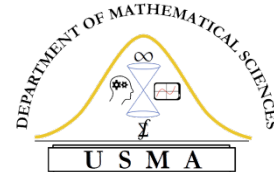
Ice Extent and Routes
1980–2100
Minimum sea ice extent and modeled future extent.
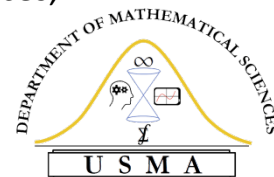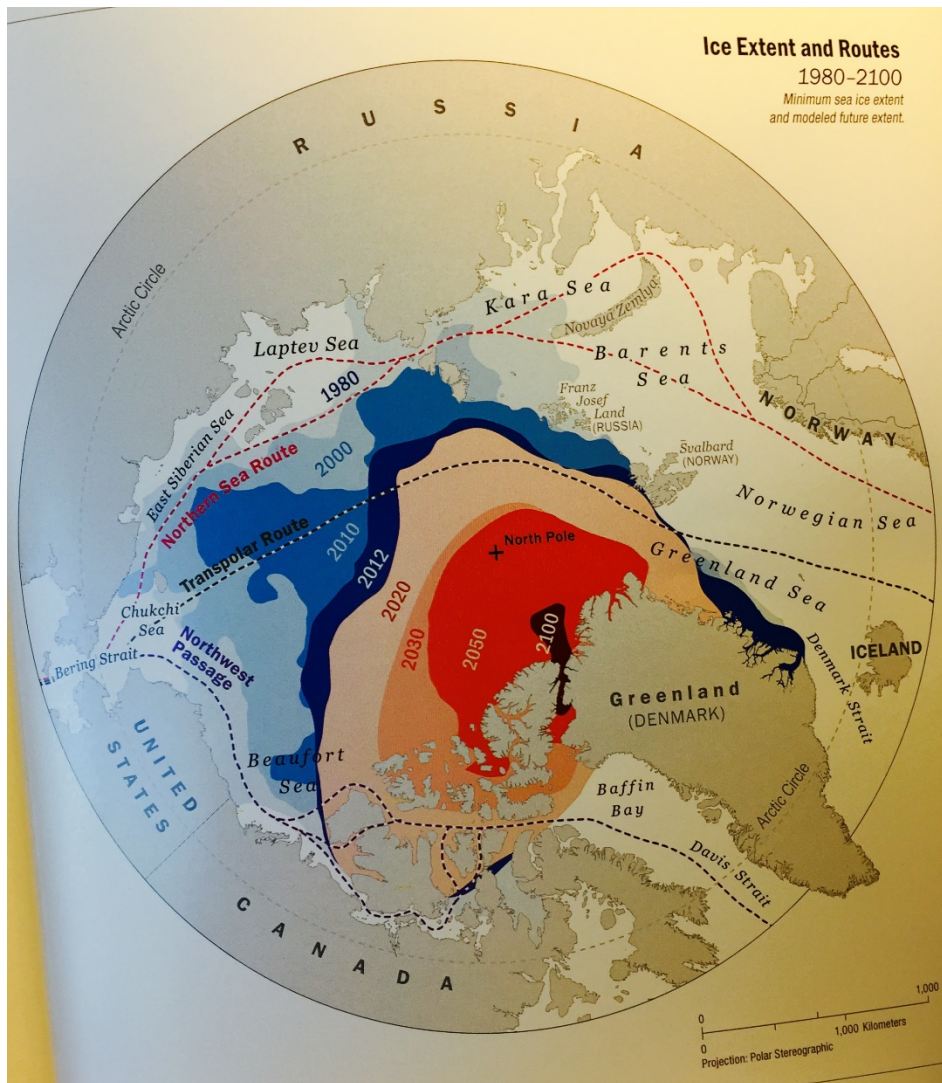
"The Arctic is becoming a region of strategic importance; its icecap is diminishing and human activity driven by economic opportunity is increasing in response to the area's growing accessibility. Some see the more accessible areas as sources of natural resources as shipping corridors become more apparent."

- National Geospatial-Intelligence Agency
(August 2015)

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Cumulative Departure Model of the Cryosphere During the Pleistocene—an Application in Computational Engineering Mathematics

Highlights of this research:

- A math model is developed to describe changes in ice volume in the cryosphere.

- Modeling the cryosphere aids in assessing future climate impacts by providing an opportunity to validate global circulation models (GCMs).

- Leveraging the dominating effects of freezing/thawing allowed for a mathematical model that can be solved exactly to investigate other components.

- Current GCM advancement trends increase complexity of various heat transport effects represented in the governing math model in cumulative form as the heat forcing function.

- Simplified models are developed whose solution can be directly compared with available data forms representing temperature and ice volume during the Pleistocene.

- By carefully integrating the Pleistocene temperature term in the solution, the well-known cumulative departure method can be resolved from this math solution.

- This simplification is shown to be a good approximation of Pleistocene ice volume for given Pleistocene temperatures.

About the Data:

The SPECMAP Project, a spectral mapping project funded by the NSF. In early 1990s, with the support of NASA, SPECMAP paleoclimate archives at NOAA/NCDC World Data Center for Paleoclimatology leveraged four archives to produce one standard chronology for oxygen isotope records. A composite of several isotopic profiles was an intentional design to eliminate 'noise' errors associated within a single isotopic record. This high resolution chronology was a composite curve derived from several isotopic records, then smoothed, filtered and tuned to known cycles of key astronomical variables.
-- NASA's Global Change Master Directory: http://gcmd.gsfc.nasa.gov/index.html
-- SPECMAP Paleoclimate Archives: http://gcmd.nasa.gov/records/GCMD_EARTH_LAND_NGDC_PALEOCL_SPECMAP.html

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U S M A
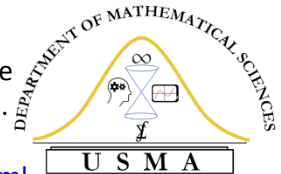
In Hromadka (2012) a mathematical analog of changes in volumetric ice in the cryosphere is given by:

$$H(t) = k \frac{I(t)}{I_0} H(t) - r \frac{dI(t)}{dt} \quad \textit{Eq. (1)*}$$

$H(t)$ = total heat into the system affecting phase change in the cryosphere

$I(t)$ = total volumetric ice in the cryosphere including glaciers, tundra ice, snow, and other forms of water subject to freeze/thaw effects

I_0 = an initial condition of $I(t)$, contemplated as a local maximum value such as during a glacial period

k = lumped parameter representing that portion of heat returned due to phase change effects, such as reflected heat due to snow, among other factors, where $0 < k < 1$
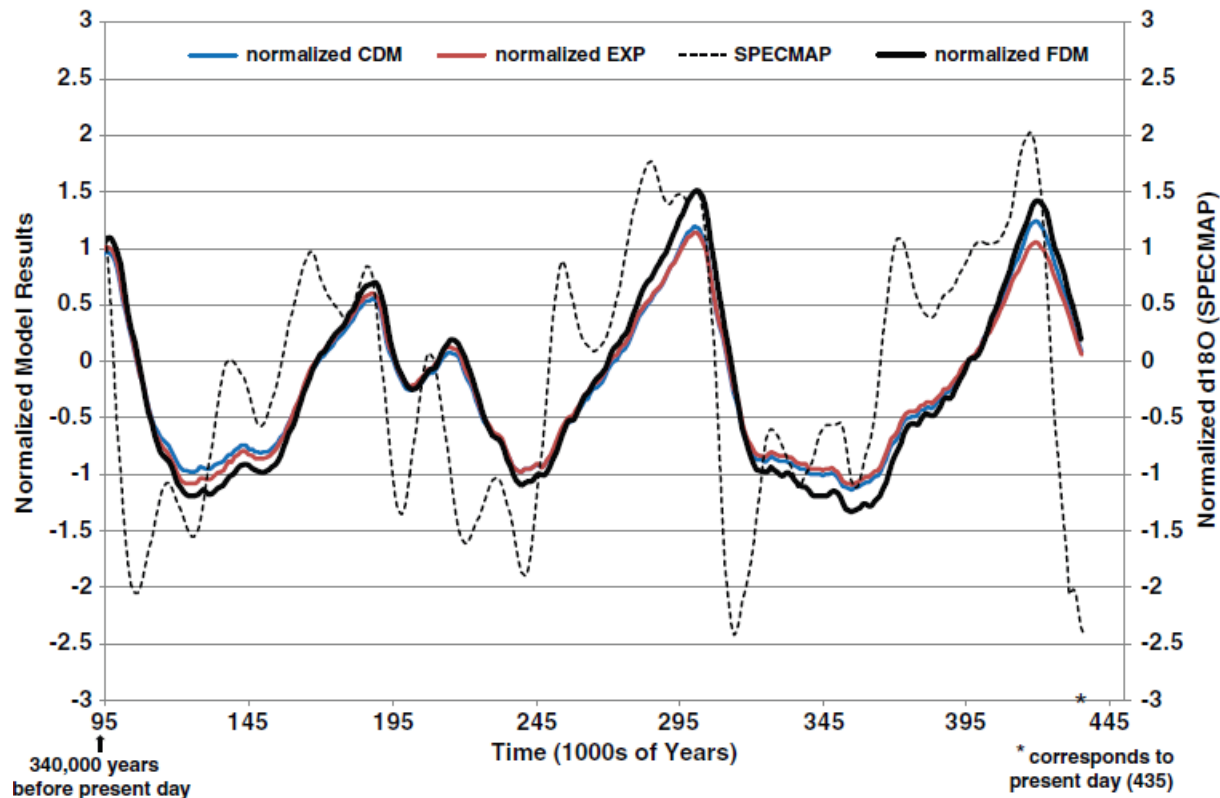
r = lumped parameter representing the area-averaged latent heat of fusion as averaged throughout the cryosphere

t = model time.

**For details regarding the underpinnings of Eq. (1), refer to Hromadka (2012).
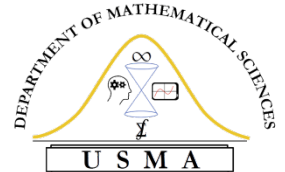
CDM: Cumulative Departure Method, used in many fields of Geoscience.

EXP: One-term Exponential version of the Simplified Exact Solution to the Differential Eqn.

SPECMAP: NASA's '**SPEC**tral **MA**pping **P**roject' containing historical climatic time series data.

FDM: Finite Difference Method used for solving Differential Eqns by approx.
them with Difference Eqns.

NOTE: The above models are suitable for undergraduate mathematical modeling application problems,
such as those found at the U.S. Military Academy at West Point, New York.

Computational Engineering Mathematics
(intersecting Applied Mathematics, Engineering, Science and Computer Science)

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What is Computational Engineering Mathematics (CEM)?

In general, computational engineering mathematics (CEM) is the development and application of computational models and simulations to solve complex physical problems arising in engineering analysis and design, as well as in natural phenomena.

The development of the cell phone you use, our understanding of Earth's climate, and the weaponry advances of the U.S. Military could not have been accomplished without CEM. Engineers use it to build new and better products, researchers use it to better understand and predict complex physical phenomena, and academics use it to advance the field itself.
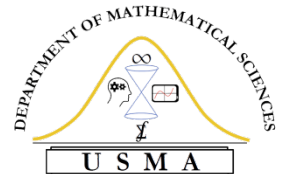
Serving Inter-Disciplinary Areas of Study

- Engineering - Social Sciences - Finance - Biological Sciences - Environmental Studies - Cyber and Big Data Applications
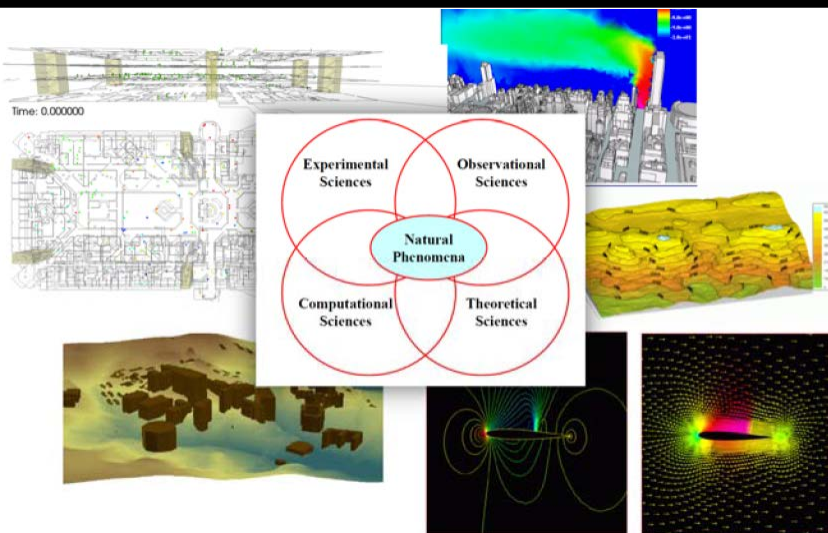

Six Local Computational Mathematics Thrusts

(1) Internships for Cadets: Six in 2016.

(2) Directed Research, Senior Thesis/Projects

(3) Journals (i.e., Applications in Computational Engineering Mathematics (ACEM))

(4) Courses, Integrative Experiences, Capstones

(5) Seminars/Conferences: Seminar, 4 Oct 16;
   Potential conference in Fall 2017

(6) Regularly fund Computational Mathematics Research and Applications (2016: CHEM, MATH, CME)

Leverage Some of West Point's Centers

- Center for Environmental and Geographical Science

- Center for Innovation and Engineering

- Center for Molecular Science

- Center for Nation Reconstruction & Capacity Development

- Center for the Study of Civil-Military Operations

- West Point Center for the Rule of Law

- Cyber Research Center

- Mathematical Sciences Center

- Network Science Center

- Operations Research Center
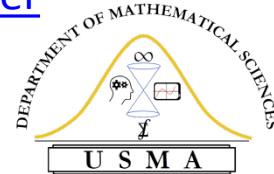
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Computational Engineering Mathematics
(intersecting Applied Mathematics, Engineering, Science and Computer Science)

Professor Carlos Brebbia: Director of the Wessex Institute of Technology. Founder and Chairman of the Computational Mechanics International Group. Editor for International Journal of Experimental Measurements. Originator of the Boundary Element Method. Research spans advanced structures to environmental problems.

Dr. Theodore V. Hromadka, II: Professor of Mathematics, U.S. Military Academy, West Point. Principal Engineer and Hydrologist for over 15 years, licenses in Civil Engineer, Geoscientist, and Geologist. 40 years professional engineering experience and 35 years of academic and research at various Institutions. Member of two American Academies.

Professor Jerry Connor: Wessex Institute of Technology representative in USA. Professor of Civil Engineering at Massachusetts Institute of Technology. Renowned for his work in software engineering and analytical techniques. Member, Institute Board of Directors.

Dr. Paolo Zannetti, QEP: President and Founder of EnviroComp Consulting Inc. and non-profit EnviroComp Institute. Leader in field of atmospheric sciences and numerical modeling. Performed scientific research in environmental sciences for over four decades.

Professor Prasada Rao, PhD: Professor in Civil and Environmental Engineering at California State University, Fullerton. Associate Director for the International Institute for Computational Engineering Mathematics. Areas: Climate Change, Surface and Subsurface Flow Modeling and Computational Mathematics.

Distinguished Colloquia in
Computational Engineering Mathematics
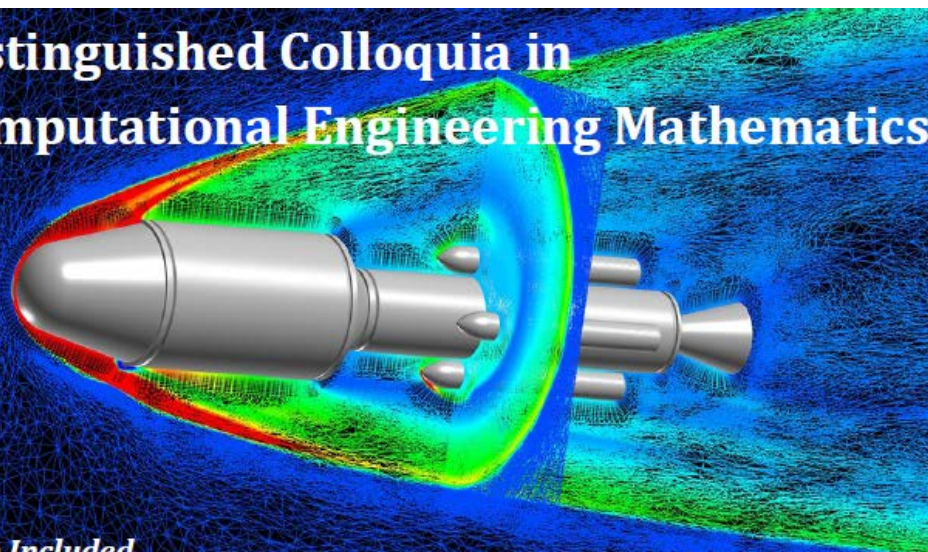
Lunch Included
Tuesday, October 4, 2016   **West Point, NY**

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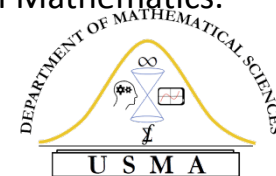*It is fitting that we are meeting in the Naupaka Room…*

There are two varieties of naupaka, one growing near the sea, the other in the mountains. Each bears what appears to be half a blossom. When placed together, they form a perfect flower.

There are several Hawaiian folk-tales about these curious blossoms. One story tells of two lovers, forever seperated by the jealous wrath of the volcano goddess Pele.

Pele desired the young man and appeared before him as a beautiful stranger; but the lovers remained inseperable. Enraged she pursued him into the mountains, hurling fiery lava after him. But Pele's gentler sisters took pity on him, and to save him from death they transformed him into the mountain naupaka. Then Pele turned on the young woman, chasing her into the sea; but again her sister goddesses intervened, changing the maiden into the beach naupaka.

And so they exist today, each blooming as a half flower, never to be united again.

NAUPAKA

NAUPAKA-KAHAKAI (beach naupaka)

The naupaka blossoms symbolize the mountains and the sea of the land of Waikoloa, seperated by the lava of the goddess Pele.

…Mathematics and the Geosciences are like the two varieties of Naupaka, one growing near the sea, the other in the mountains. When placed together, they form a perfect flower!

- Common interests and opportunities in the study of the earth's environment, physical structure, substance, history, and the processes that act on it.

- Threads of analyses for Computational Engineering Mathematics are partnership opportunities between the disciplines.

- Interested in YOUR efforts that pertain to or can be informed by CEM approaches.

- Consider West Point a partner in the mathematics that support modeling of large-scale applications and analysis of risk and reliability in the environmental engineering and geosciences.

- Geologists *ROCK*!

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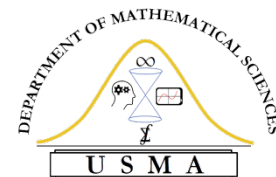
THANK YOU!

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